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Influence of Holdups and Feed on the Transient Process of Pulse Cascades

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In isotope separation pulse cascades are non-conventional and transient processes take place at each stage and in the whole cascade. A numerical method is developed to study the transient process of reaching steady state and applied to square pulse cascades to identify the majors of influence. Without feed and withdrawals, the main factors influencing the transient process are the cascade length and the ratio of the centrifuge holdup to the pipe holdup. With feed and withdrawals, the factors influencing the transient process include the feed, the centrifuge holdup, the pipe holdup, and the cascade length.

Keywords centrifuge cascade; isotope separation; pulse cascade; transient process

INTRODUCTION

Non-conventional cascades may have advantageous separation properties over conventional cascades in some cases of isotope separations, and may find their practical applications in enriching middle components, or components of small concentrations, or quickly obtaining a highly enriched component. Purging cascades with additional feed (1) and non-stationary cascades (2–4) are examples of non-conventional cascades. Pulse cascade is a new concept of non-conventional cascades, which was first proposed recently (5). The basic idea of the pulse cascade is to make use of the property of centrifuges that the overall separation factor increases with the decrease of the feed.

Pulse cascades work in a pulsant manner like a sequence of pulses. A pulse consists of two operation states—the closed state and the open state. In the closed state, every stage is isolated with each other. The centrifuges of a stage are working with the recirculating flows from the head pipe and the tail pipe of this stage as the feed. During the open state, the stages are connected in a way as in an ordinary conventional cascade, and the materials are exchanged between the neighboring stages. Following the open state, the closed state of the next pulse begins. Because of the manner of the alternating operation between the closed and the open states, there exist no steady and continuous flows in pulse cascades. Therefore, unlike conventional cascades, it is the holdups in the head and tail pipes that determine the shape of a pulse cascade, other than the flows in the head and tail pipes. For example, an ideal pulse cascade can be created by properly prescribing the holdups in the centrifuges and the pipes (5). The results show that to obtain the same concentration, using a pulse cascade needs much shorter cascade length than a conventional cascade. This implies a number of applications, for example, using a small pulse cascade with only one centrifuge at each stage can achieve a degree of enrichment of a large conventional cascade.

Because the concept of pulse cascades was presented not long ago, it has not been studied thoroughly. One task of this paper is to develop a numerical method for analyzing pulse cascades, because the analytical method used in (5) for the ideal cascade is not applicable in analyzing general cascades. Although it seems that in some cases a pulse cascade demonstrates stronger enrichment capabilities, such as producing a higher concentration with a shorter cascade length as in the above mentioned example, the properties of pulse cascades in many aspects are unknown. Since transient processes are unavoidable in pulse cascades due to the pulsant working manner, an important aspect is how the transient processes of a pulse cascade are influenced by the many quantities, for example, the holdups in the centrifuges, the head pipes and tail pipes of every stage, as well as the feed and withdrawals of the cascade. It is expected, based on the experience of research on conventional cascades, that their magnitudes would have important influences on the transient process from an initial state to the steady state and hence on the separation performance. So another task is to study the transient processes in pulse cascades to reveal the effects of the holdups and the feed and identify the major factors of influences. The study here focuses on the principles of pulse cascades. The engineering issues for implementation of pulse cascades are not considered here.

In Section 2, the operation of pulse cascades are explained briefly and the method for analysis is presented. The model pulse cascade used in the study, the so-called square pulse cascade, is introduced in Section 3. Square

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cascades with and without feed and withdrawals are investigated, respectively in Sections 4 and 5. The conclusions are drawn in Section 6.

THE OPERATION PROCESSES AND ANALYSIS METHOD

The details of the operation of pulse cascades are described in (5). Figures 1 and 2 are the sketches of the closed state and the open state of a pulse cascade. Here H_n, H'_n, and H''_n are the holdup in the centrifuges, the head pipe and the tail pipe at the n-th stage, which are called the "centrifuge holdup," the "head holdup," and the "tail holdup" for short respectively. $\overline{C}_{i,n}$, $\overline{C}'_{i,n}$, and $\overline{C}''_{i,n}$ are the average concentrations of the i-th component in above corresponding centrifuges and pipes. $V'_{1,n}$, $V'_{2,n}$, $V''_{1,n}$, and $V''_{2,n}$ are the valves before and after the head pipe and the tail pipe at the n-th stage, respectively.

One closed state and one open state constitute a pulse. In a closed state, the valves $V'_{1,n}$ and $V''_{1,n}$ are open, and the valves $V'_{2,n}$ and $V''_{2,n}$ are closed. The mixtures in the head and tail pipes at the n-th stage are fed back to form the feed of the centrifuges of this stage. These flows are the "recirculating flows" of the n-th stage through the recirculating pipes where the valves $V'_{3,n}$ and $V''_{3,n}$ are open, as shown in Fig. 1. They are denoted as L'_n for up-recirculating flows and L''_n for down-recirculating flows. Because the valves $V'_{2,n-1}$, $V''_{2,n}$, $V'_{2,n}$, and $V''_{2,n+1}$ are closed, the n-th stage is isolated with its two neighboring stages, i.e., the $(n-1)$-th and the $(n+1)$-th stages. Each stage undergoes a transient process until some criterion is satisfied to start the open state. In an open state, the valves $V'_{1,n}$, $V''_{1,n}$, $V'_{3,n}$, and $V''_{3,n}$ are closed, and the valves $V'_{2,n}$ and $V''_{2,n}$ are open. The mixtures in the head pipe of the $(n-1)$-th stage and the tail pipe of the $(n+1)$-th stage enter the centrifuges of the n-th stage and mix with the mixture inside. At the same time, the feed F_n is introduced into the centrifuges, and the withdrawals P_n and W_n are respectively withdrawn from the head pipe and the tail pipe at the n-th stage simultaneously in the open state, as shown in Fig. 2. So it is obvious that the holdups in the pipes of every stage must be no smaller than the corresponding withdrawals, $H'_n \geq P_n$ and $H''_n \geq W_n$. Of course, the feeds and the withdrawals at most stages are zero. Note that the feeds and withdrawals are not continuous flows, but are intermittently supplied or extracted quantities of material in step with pulses.

The time between two successive "closed" or "open" states is defined as a pulse period (5), during which the cascade experiences one closed state and one open state, which are referred to as the first phase and the second phase respectively for convenience. Now another transient process can be identified, which is the transient process of the cascade. After a number of pulses, one can expect that the distributions of components along the cascade would reach a steady state.

Suppose that the total number of stages of a pulse cascade, which has a feed and two withdrawals, is N. The feed F is supplied at the stage of number N_F. The product P and the waste W are withdrawn at the last and the first stages respectively. The isotopic mixture has N_C components, which are indexed as 1, 2,..., N_C respectively, according to their molar weights in an ascending order. The feed concentration of the i-th component is denoted as $C_{i,F}$. The head holdup and the tail holdup are related with the feed and withdrawls in the pulse cascade as in the following:

$$H'_n - H''_{n+1} = -W, \quad (1 \leq n < N_F), \tag{1}$$

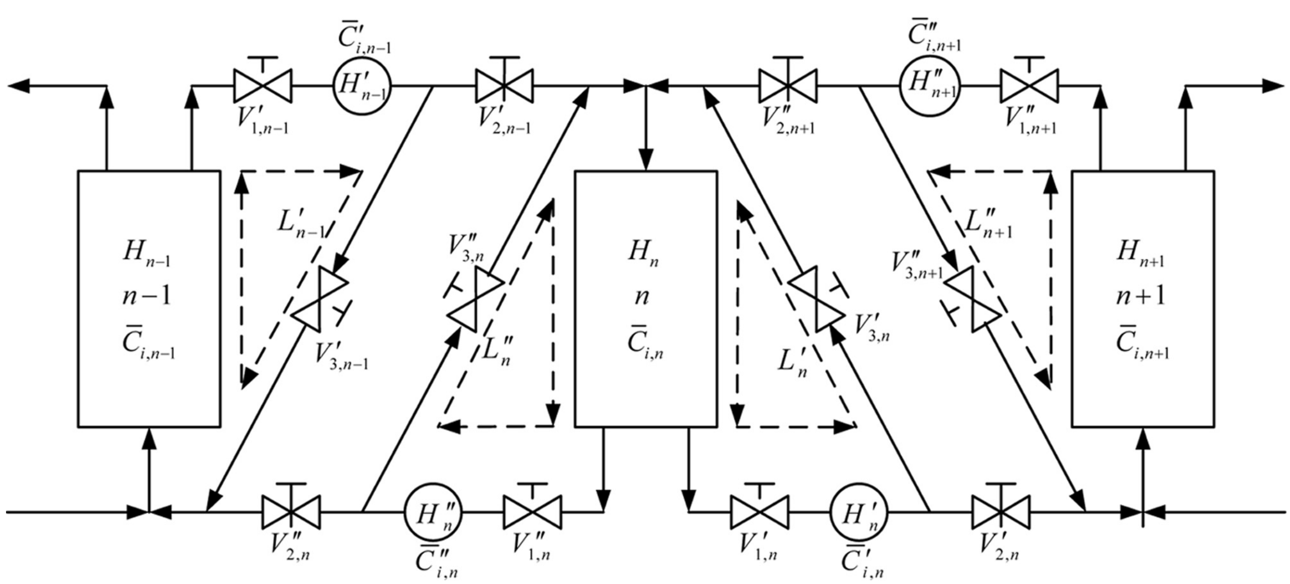

FIG. 1. The sketch of the closed state of a pulse cascade.

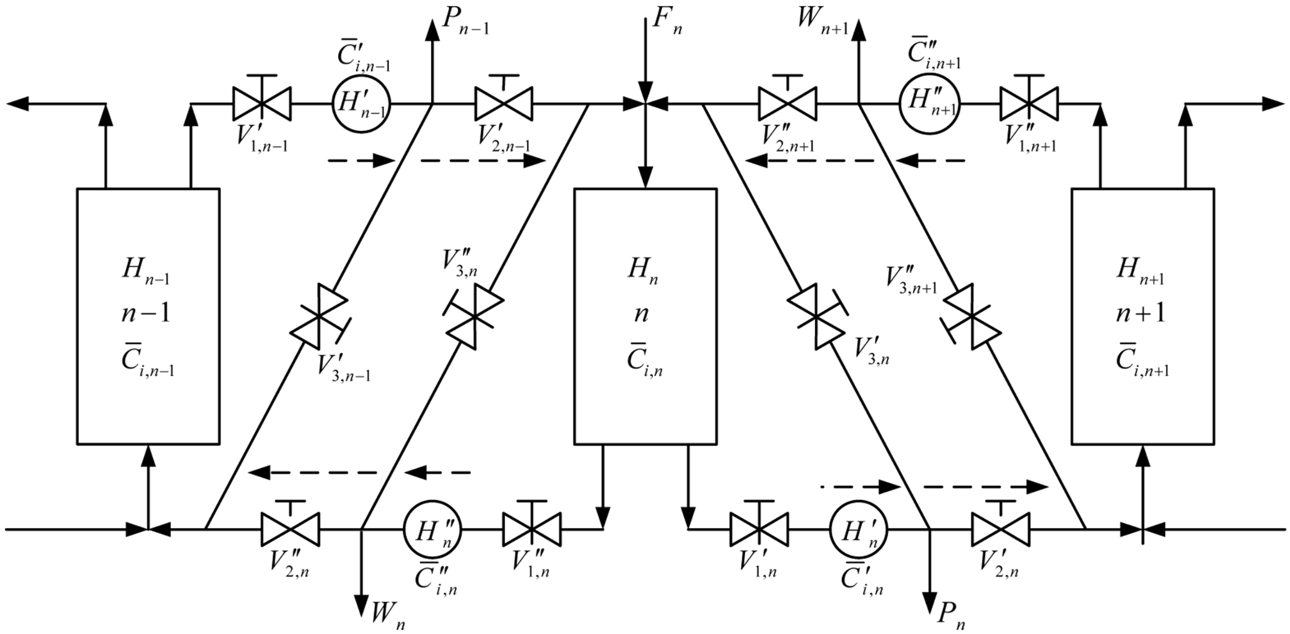

FIG. 2. The sketch of the open state of a pulse cascade.

$$H'_n - H''_{n+1} = P, \quad (N_F \le n < N). \tag{2}$$

So in the second phase of a pulse, there are holdup fluxes from the feed stage to the two end stages. They are called the “net holdup transports” which are equal to P or W in different sections of the cascade, like the net flows in conventional cascades.

In the first phase, every stage in the pulse cascade is isolated with each other. The time-dependent equations for mass conservation are given by (cf (6,7)):

$$\frac{\partial H_n \overline{C}_{i,n}}{\partial t} = L'_n(C'^{a}_{i,n} - C'_{i,n}) + L''_n(C''^{a}_{i,n} - C''_{i,n}), \tag{3}$$

$$\frac{\partial H'_n \overline{C}'_{i,n}}{\partial t} = L'_n(C'_{i,n} - C'_{a_{i,n}}), \tag{4}$$

$$\frac{\partial H''_n \overline{C}''_{i,n}}{\partial t} = L''_n(C''_{i,n} - C''^{a}_{i,n}), \tag{5}$$

where $C'_{i,n}$ and $C'^{a}_{i,n}$ are the concentrations of the i-th component before and after the head pipe at the n-th stage, respectively, and $C''_{i,n}$, $C''^{a}_{i,n}$ are those for the tail pipe. The average concentrations $\overline{C}_{i,n}$, $\overline{C}'_{i,n}$ and $\overline{C}''_{i,n}$ are approximated by:

$$\overline{C}_{i,n} = \frac{L'_n}{L'_n + L''_n}\frac{C'_{i,n} + C'^{a}_{i,n}}{2} + \frac{L''_n}{L'_n + L''_n}\frac{C''_{i,n} + C''^{a}_{i,n}}{2}, \tag{6}$$

$$\overline{C}'_{i,n} = \frac{C'_{i,n} + C'^{a}_{i,n}}{2}, \tag{7}$$

$$\overline{C}''_{i,n} = \frac{C''_{i,n} + C''^{a}_{i,n}}{2}. \tag{8}$$

Together with Eq. (9) in the following,

$$\gamma_0^{M_j - M_i} = \frac{C'_i}{C'_j} \Big/ \frac{C''_i}{C''_j}. \tag{9}$$

which is the property of separators using the kinetic method for isotope separation, Eqs. (3)–(5) can be discretized and changed to a set of difference equations, and numerically solved by a time-marching method with the Q-iteration (6,7). If the following equation

$$\max_i \left(\frac{\left| C'^{a}_{i,n} - C'_{i,n} \right|}{C'_{i,n}}, \frac{\left| C''^{a}_{i,n} - C''_{i,n} \right|}{C''_{i,n}} \right) \le \varepsilon_1 \tag{10}$$

holds, where ε_1 is a small given number, the transient process is over, and the steady state at the n-th stage is reached. When the transient processes at all stages are over, the second phase begins.

Use $\overline{C}^{m}_{i,n}$, $\overline{C}'^{m}_{i,n}$, and $\overline{C}''^{m}_{i,n}$ to denote the concentrations of the i-th component in the centrifuges, the head pipe, and the tail pipe at the n-th stage at the end of the first phase of the m-th pulse. In the second phase, at the n-th stage, the mixtures in the head pipe of the $(n-1)$-th stage, the tail pipe of the $(n+1)$-th stage, and the centrifuges at the n-th stage mix, and are the material to be separated in the next coming pulse. During this phase the main process taking place is the material exchange between the neighboring stages and uses little time, therefore, the separation effect can be ignored. So it is reasonable to assume that the initial state of the $(m+1)$-th pulse is that the concentrations of the i-th component in the centrifuges, the head pipe, and the tail pipe at the n-th stage are the same, which are denoted as $C_{i,n}^{m+1}$. Then the equation at the end of the second phase is:

$$H_n\overline{C}_{i,n}^{m} + \left(H'_{n-1} - P_{n-1}\right)\overline{C}_{i,n-1}^{\prime m} + \left(H''_{n+1} - W_{n+1}\right)\overline{C}_{i,n+1}^{\prime\prime m} + F_n C_{i,n}^{F,m} = \left(H_n + H'_n + H''_n\right)C_{i,n}^{m+1}. \quad (11)$$

In the first phase of the $(m+1)$-th pulse, the operation process in the pulse cascade is also described by Eqs. (3)–(5). After each stage reaches steady state again, the following relationship of the concentrations holds:

$$H_n\overline{C}_{i,n}^{m} + \left(H'_{n-1} - P_{n-1}\right)\overline{C}_{i,n-1}^{\prime m} + \left(H''_{n+1} - W_{n+1}\right)\overline{C}_{i,n+1}^{\prime\prime m} + F_n C_{i,n}^{F,m} = H_n\overline{C}_{i,n}^{m+1} + H'_n\overline{C}_{i,n}^{\prime m+1} + H''_n\overline{C}_{i,n}^{\prime\prime m+1}. \quad (12)$$

When the concentrations in the centrifuges, the head pipe, and the tail pipe of every stage do not change at the end of the first phase of two successive pulses, as judged by Eq. (13), where ε_2 is a small given number, the whole pulse cascade reaches its steady state.

$$\max_{i,n}\left(\frac{\left|\overline{C}_{i,n}^{m+1} - \overline{C}_{i,n}^{m}\right|}{\overline{C}_{i,n}^{m}}, \frac{\left|\overline{C}_{i,n}^{\prime m+1} - \overline{C}_{i,n}^{\prime m}\right|}{\overline{C}' m_{i,n}}, \frac{\left|\overline{C}_{i,n}^{\prime\prime m+1} - \overline{C}_{i,n}^{\prime\prime m}\right|}{\overline{C}_{i,n}^{\prime\prime m}}\right) \le \varepsilon_2. \quad (13)$$

SQUARE PULSE CASCADES

Use H_n^P to denote the total holdup in the head and tail pipes at the n-th stage, called the "pipe holdup," and h_n the ratio of the head holdup to the pipe holdup at the n-th stage, called the "holdup cut." Then, the holdups in the head and tail pipes can be expressed by H_n^P and h_n.

$$H_n^P = H'_n + H''_n, \quad (14)$$

$$h_n = \frac{H'_n}{H'_n + H''_n} = H'_n/H_n^P. \quad (15)$$

Here we refer to a pulse cascade whose pipe holdups of all stages are equal as a "square pulse cascade", that is,

$$H_n^P = H^P = \text{const.} \quad (16)$$

In the second phase of one pulse, a square pulse cascade requires "recirculating holdups" at the two end stages, like the "recirculating flows" in a conventional cascade, which means that part of the head holdup of the last stage needs to be fed back to the feed of this stage, and part of the tail holdup of the first stage back to the feed of this stage, too. Or the material balances at the two end stages cannot be maintained, and the square pulse cascade cannot be formed.

Here in the following study, we consider only square cascades to avoid introducing too many variables and making things complicated before having gained some primary knowledge about pulse cascades. Furthermore, square cascades are more frequently adopted in separation of isotopes of different elements.

SQUARE PULSE CASCADES WITHOUT FEED AND WITHDRAWALS

To investigate the influence of the holdups in pulse cascades, first consider pulse cascades without feed and withdrawals, i.e., $F = P = W = 0$. Such cascades are able to obtain a high concentration at the product end for the lightest component and at the waste end for the heaviest component. Suppose that the pulse cascades here are all square pulse cascades in which the centrifuge holdups of all stages are constant:

$$H_n = H = \text{const.} \quad (17)$$

The holdup cut at the first stage $h_1 = 0.5$, the ratio of the centrifuge holdup to the pipe holdup of every stage $H/H^P = 1/2$, the recirculating flow rates in the first phase $L'_n = L''_n$, and the overall separation factors at all stages $\gamma_0 = 1.2$. Note that all dimensional quantities in the above are specified relative to one unit of the corresponding quantities. For example, if one unit of material is defined as 5 moles, then $H = 1$ means the holdup in the centrifuges of a stage is 5 moles. The recirculating flow rates also depend on the unit of time, e.g., if the unit of time is 2 minutes, $L' = 1$ means that the up-recirculating flow rate is 1 unit of material divided by 1 unit of time, that is, 5 moles per 2 minutes or 2.5 moles per minute. The choice of units is entirely up to the convenience of researchers. As mentioned in (8), the separation factor of a centrifuge increases monotonously with the decrease of the feed flow. To keep the separation factor all the same for all centrifuges, it demands that the feed flow rates are identical for all centrifuges. In all the following calculations, the unit of the time is taken to be a minute and the recirculating flow rates in the first phase are assumed to be numerically equal to the centrifuge holdup: $L'_n = L''_n = H_n$. The isotopic mixture to be separated is WF_6, which has five components: $^{180}WF_6$, $^{182}WF_6$, $^{183}WF_6$, $^{184}WF_6$, and $^{186}WF_6$, whose natural concentrations are 0.0012, 0.265, 0.1431, 0.3064, and 0.2843, respectively.

At the beginning of the calculation, it is assumed that the centrifuges, the head pipe and the tail pipe of every stage in the pulse cascade are all filled with the feed mixture:

$$\overline{C}^0_{i,n} = \overline{C}'^0_{i,n} = \overline{C}''^0_{i,n} = C_{i,F}, \quad (i = 1, 2, ..., N_C; n = 1, 2, ..., N). \tag{18}$$

The criteria in the calculation are taken as follows:

$$\varepsilon_1 = 10^{-7}, \quad \varepsilon_2 = 10^{-6}. \tag{19}$$

In practice, ε_1 and ε_2 do not have to be so small.

First, to understand the effects of the total holdup in cascades, i.e.,

$$\sum_n (H_n + H_n^P),$$

the following two cases are investigated:

a. The total number of stages is fixed at $N = 6$, but the pipe holdup H^P are different;

b. The total number of stages N and the pipe holdup H^P are both different, but the total centrifuge holdup and the total pipe holdup are fixed at:

$$\sum_n H_n = 60, \quad \sum_n H_n^P = 120. \tag{20}$$

The number of pulses needed to reach the steady state of cascade, or in other words the final pulse number N_{FP}, and the total time consumed from the first pulse to the final pulse, or in short the transient time T_{FP}, are two factors indicating how fast the product can be obtained. Obviously the smaller they are, the better. The final pulse numbers and the transient times in the above two cases are presented in Figs. 3 and 4. It can be seen from Case a that the final

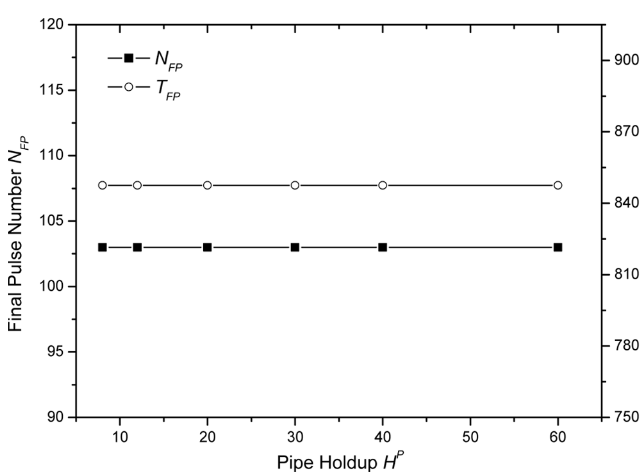

FIG. 3. The final pulse number and transient time vs. the pipe holdup with fixed cascade length.

pulse number and the transient time do not change with the change of the total holdup in cascade, but from Case b the final pulse number and the transient time increase with the increase of the total number of stages when the total centrifuge holdup and the total pipe holdup are fixed for different pulse cascades.

This phenomenon can be explained as follows. Take the first component as an example. Because the initial states in all stages in the pulse cascade are identical, at the end of the first phase of the first pulse, the concentrations in the centrifuges at all stages $\overline{C}^1_{1,n}$ are the same, which are equal to $C_{1,F}$. The concentrations in the head pipes at all stages $\overline{C}'^1_{1,n}$ and those in the tail pipes $\overline{C}''^1_{1,n}$ are respectively the same too. In the second phase, the material transportation takes place in the cascade as described in the second section. Because the material entering the first stage consists of two tails, and that entering the last stage consists of two heads, only $\overline{C}^2_{1,1}$ and $\overline{C}^2_{1,N}$ are different from $C_{1,F}$ at the end of the first phase in the second pulse: $\overline{C}^2_{1,1}$ is smaller and $\overline{C}^2_{1,N}$ is larger than $C_{1,F}$. After three pulses, one can imagine that the two stages at each end of the cascade have concentration changes. That is, $\overline{C}^3_{1,1}$, $\overline{C}^3_{1,2}$ are smaller than $\overline{C}^2_{1,1}$, $\overline{C}^2_{1,2}$, and $\overline{C}^3_{1,N}$, $\overline{C}^3_{1,N-1}$ are larger than $\overline{C}^2_{1,N}$, $\overline{C}^2_{1,N-1}$, respectively. In this way, the concentration changes starting at the two ends spread toward the interior stages of the cascade stage by stage through pulses. After several pulses, the concentration distribution of the first component becomes monotonically increasing along the cascade. For a pulse cascade with a total of six stages for separating the WF_6 mixture, the trendlines of the head concentrations of the first component at all stages with respect to the pulse number are shown in Fig. 5, which is consistent with the above analysis. Since the stage setups in different cascades are identical, the time of a pulse changes little from one cascade to another. So in Fig. 4 the transient times T_{FP} behaves in a similar way as the final pulse numbers N_{FP}.

In summary, the length of the pulse cascade is an important factor influencing the transient process of cascade. The longer the cascade is, the more pulses are needed for the concentration changes caused by two ends to spread to the interior, and therefore, the larger the final pulse number is and the longer the transient time is.

Secondly, it is interesting to know how the transient process is affected by the holdups H and H^P. One may naturally expect that it is the total holdup of a stage $H + H^P$ that affects the transient process. But in fact it is that the ratio of the centrifuge holdup to the pipe holdup H/H^P. As shown in Fig.6, which are the calculation results of the cascades with fixed length of $N = 6$, the final pulse number increases linearly with the increase of the ratio H/H^P. In this figure, the two curves with solid dots, one for fixed H and the other for fixed H^P, are overlapping and hardly distinguishable from each other.

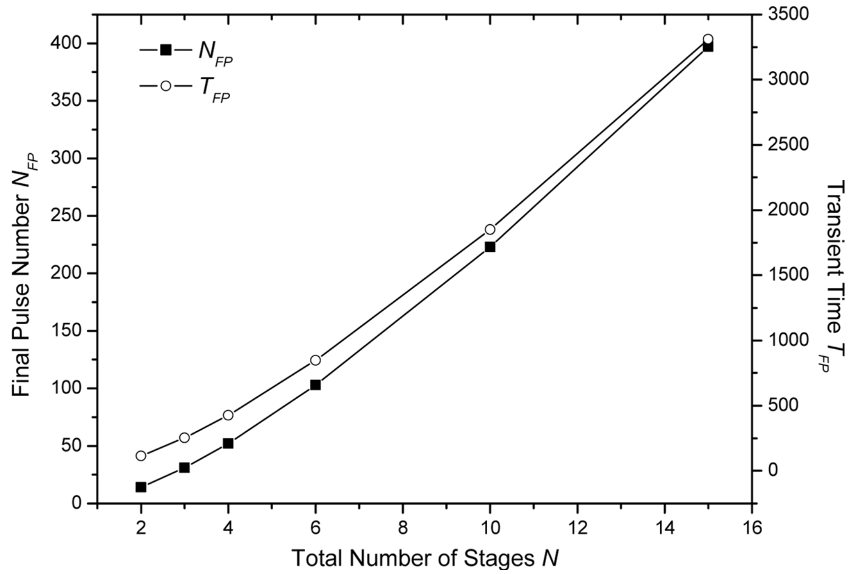

FIG. 4. The final pulse number and transient time vs. the total number of stages with fixed total holdup in cascade.

This can be explained in the following. In the second phase of the m-th pulse, the mixture in the n-th stage mixing with the mixtures from their neighboring stages is the material to be separated in the next coming pulse, and has the concentration given by the following equation:

$$C_{i,n}^{m+1} = \frac{H'_{n-1}\overline{C}'^{m}_{i,n-1} + H_n\overline{C}^{m}_{i,n} + H''_{n+1}\overline{C}''^{m}_{i,n+1}}{H'_n + H_n + H''_n}. \qquad (21)$$

Here since $h_1 = 0.5$, H'_n and H''_n are equal to half of H_n^P. Besides, because $L'_n = L''_n$, it can be deduced that $\overline{C}_{i,n}^{m+1} = C_{i,n}^{m+1}$ from Eqs. (6) and (10). Then the following equation is obtained:

$$\left|\overline{C}_{i,n}^{m+1} - \overline{C}_{i,n}^{m}\right| = \frac{\left|\overline{C}'^{m}_{i,n-1} + \overline{C}''^{m}_{i,n+1} - 2\overline{C}^{m}_{i,n}\right|}{2\left(H_n/H_n^P + 1\right)}. \qquad (22)$$

It can be seen that, if H_n is large relative to H_n^P, the absolute value of the difference between $\overline{C}_{i,n}^{m+1}$ and $\overline{C}_{i,n}^{m}$, i.e., $\left|\overline{C}_{i,n}^{m+1} - \overline{C}_{i,n}^{m}\right|$ is small. This means that one pulse can only cause little change in concentrations. Then, the concentration needs more pulses to reach the steady state. So in the cases when the centrifuge holdup is large relative to the pipe holdup, the final pulse number is large.

On the contrary, the transient time decreases with the increase of the ratio H/H^P, as the two curves with hollow dots shown in Fig. 6. This is somewhat surprising. The reason is that the time of a pulse mainly depends on the relative magnitudes of the pipe holdup to the recirculating flow rates. When the pipe holdup is relatively larger, a pulse spends longer time. Here the ratio of the recirculating flow rates to the centrifuge holdups is fixed, so when the centrifuge holdup is large relatively to the pipe holdup, the time consumed in a pulse is short, which leads to short transient time.

To conclude the above results and analyses, in the situation without feed and withdrawals, the concentration changes appear first at the two ends of the cascade, and the final pulse number and the transient time depend on the spread of the concentration changes caused by the two ends. The total number of stages and the ratio H/H^P are the crucial factors of influencing the transient process. If the ratio is constant, the primary factor is the cascade length, neither the centrifuge holdup and the pipe holdup of every stage, nor the total holdup of the cascade; if the

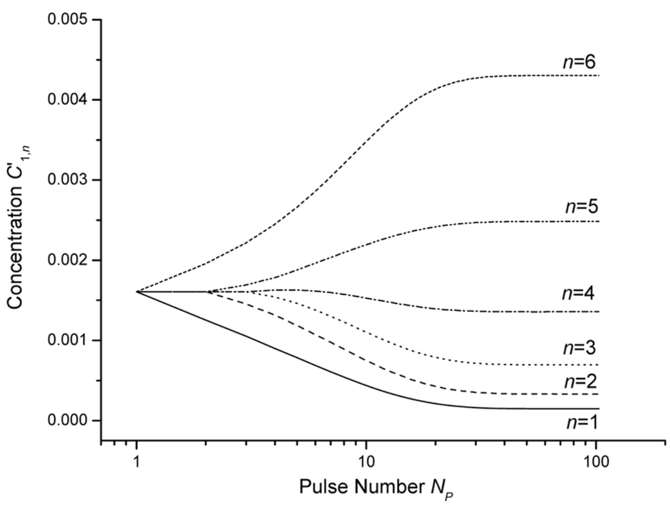

FIG. 5. The head concentration of the 1st component at each stage in the pulse cascade without feed and withdrawals.

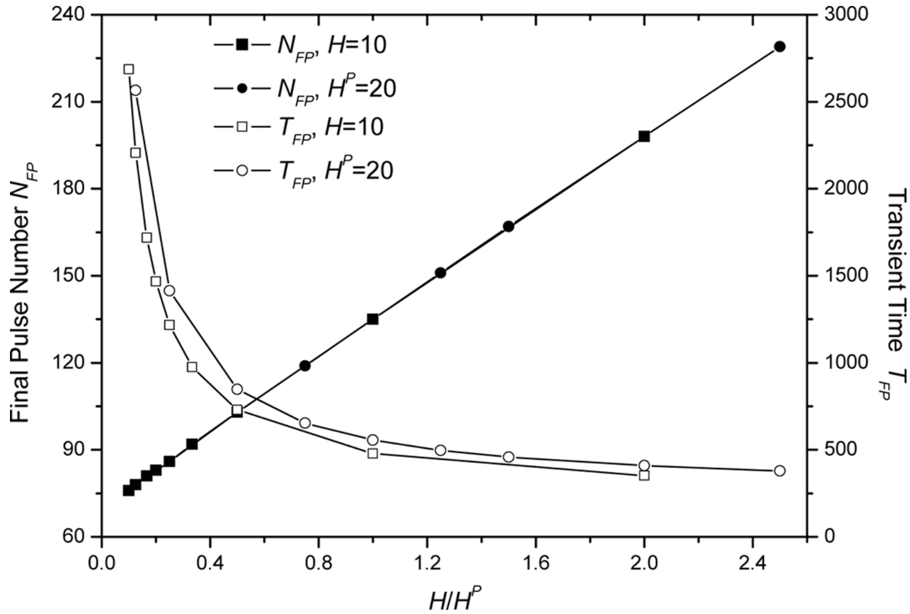

FIG. 6. The final pulse number and transient time as a function of the ratio of the centrifuge holdup to the pipe holdup.

cascade length is fixed, the primary factor is the ratio H/H^P. This suggests that for a pulse cascade, it is important to chose an appropriately moderate ratio H/H^P.

SQUARE PULSE CASCADES WITH FEED AND WITHDRAWALS

In this section, square pulse cascades with a feed and two withdrawals are utilized to separate the WF_6 mixture. The total number of stages $N=6$, the centrifuge holdup for each stage $H=10$, the pipe holdup for each stage $H^P=20$, and the holdup cut at the first stage $h_1=0.5$. The feed $F=1$ is supplied into the cascade at the stage $N_F=3$, and the product $P=0.2$ is withdrawn from the head pipe at the N-th stage. The trendlines for the head concentrations of the first component at all stages with respect to the pulse number are given in Fig. 7 for this case.

At the stages from stage 1 to stage 4, the head concentrations of the first component decrease at the beginning, but then increase till the steady state is reached. At the 5-th and 6-th stages, compared with Fig. 5, the head concentrations of the first component clearly appear to increase at two steps, the first of which begins at pulse number 1, the same as in Fig. 5, and the second begins at about pulse number 13.

This is understood as follows. Looking at the two curves for $C'_{1.1}$ at $F=0$ and $F=1$, as shown in Fig. 8. The concentration in the first stage decreases gradually at first as in the case without feed and withdrawals, which is dominated by the spread of the concentration changes caused by the two ends. After three pulses, the influence of the feed arrives and causes $C'_{1.1}$ to decrease slower than in the case of $F=0$. Eventually, after stopping decreasing the concentration begins to increase at pulse number 13, other than to keep unchanged. The situations at the other stages are similar, as shown in Fig. 8. It is observed that reaching the steady state needs more number of pulses for nonzero F.

The four factors considered here in determining the transient process of reaching steady state in a pulse cascade with feed and withdrawals are the cascade length, the feed, the centrifuge holdup, and the pipe holdup. The variations of the final pulse numbers and the transient times with

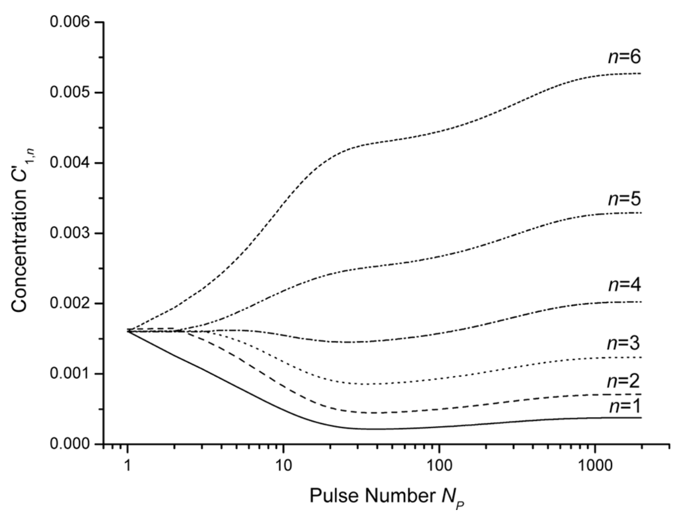

FIG. 7. The head concentration of the 1st component at each stage in the pulse cascade with feed and withdrawals.

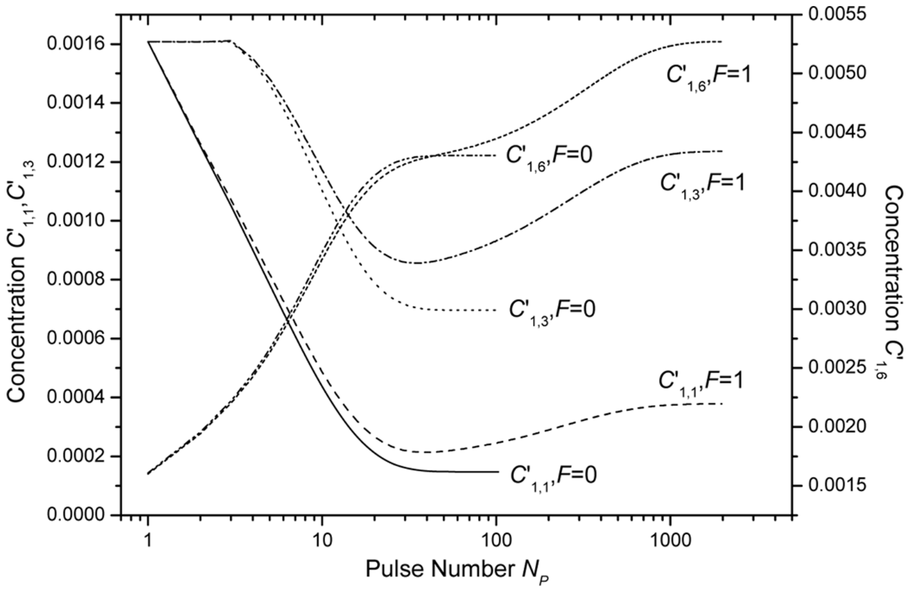

FIG. 8. The head concentration of the 1st component at the first and the last stage in pulse cascades with and without feed and withdrawals.

respect to those parameters are presented in Figures 9 to 12, respectively.

According to these figures, the following conclusions are drawn:

a. The longer the pulse cascade is, the larger the final pulse number is, and the longer the transient time is;
b. The smaller the feed is, the larger the final pulse number is, and the longer the transient time is;
c. The larger the centrifuge holdup is, the larger the final pulse number is, but the shorter the transient time is;
d. The larger the pipe holdup is, the larger the final pulse number is, and the longer the transient time is.

Take the stripping section for an example. Since $h_1 = 0.5$, there are two cases for the distributions of the holdups in the head and tail pipes deduced from Eq. (1):

a. $H'_n = H''_n = H^P_n/2,\ H'_{n-1} = H^P_n/2 - W,\ H'_{n-1} = H^P_n/2 + W$;

b. $H'_n = H^P_n/2 - W,\ H'_{n+1} = H^P_n/2 + W,\ H'_{n-1} = H''_{n+1} = H^P_n/2.$

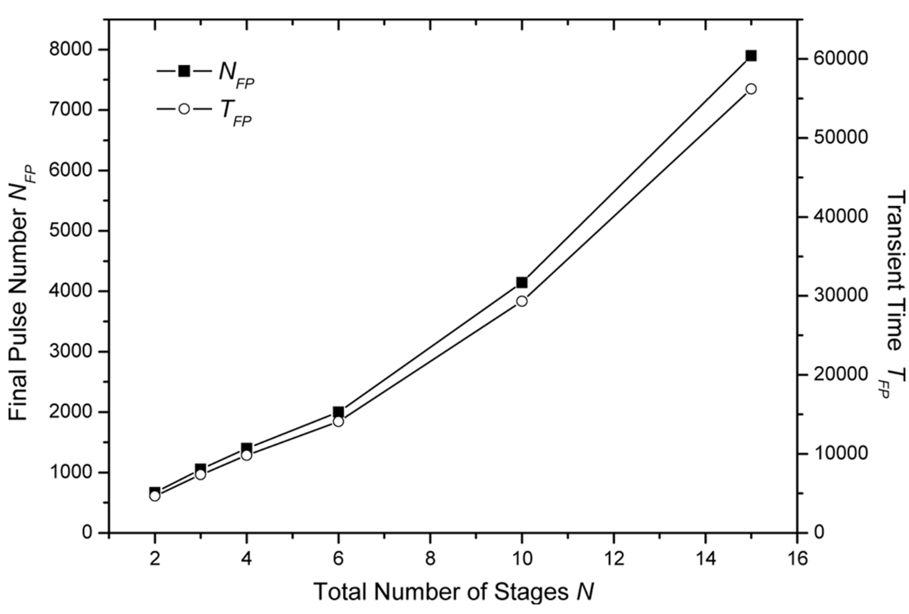

FIG. 9. The final pulse number and transient time for different cascade lengths ($N_F = 2$, $H = 10$, $H^P = 20$, $F = 1$, $P = 0.2$).

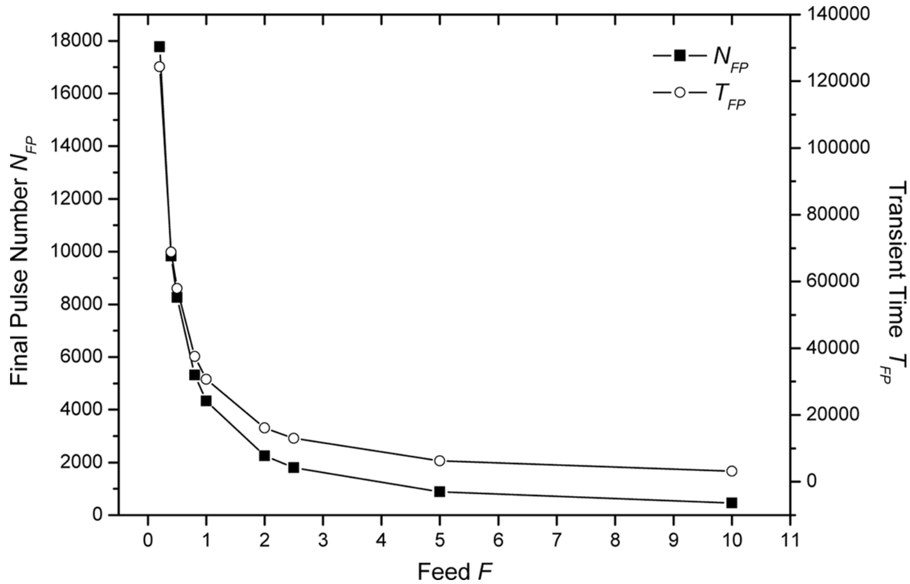

FIG. 10. The final pulse number and transient time for different feeds ($N=11$, $N_F=5$, $H=10$, $H^P=20$, $P/F=0.2$).

Using the similar method mentioned in section 4, the absolute value of the difference between $\overline{C}_{i,n}^{m+1}$ and $\overline{C}_{i,n}^{m}$, i.e.,

$$\left|\overline{C}_{i,n}^{m+1}-\overline{C}_{i,n}^{m}\right| \le \frac{H_n^P/W}{2\left(H_n/W+H_n^P/W\right)}\left|\overline{C}'^{m}_{i,n-1}+\overline{C}''^{m}_{i,n+1}-2\overline{C}^{m}_{i,n}\right| + \frac{\left|\overline{C}'^{m}_{i,n}-\overline{C}''^{m}_{i,n}\right|}{H_n/W+H_n^P/W} \tag{23}$$

for Case a, and

$$\left|\overline{C}_{i,n}^{m+1}-\overline{C}_{i,n}^{m}\right| \le \frac{H_n^P/W}{2\left(H_n/W+H_n^P/W\right)}\left|\overline{C}'^{m}_{i,n-1}+\overline{C}''^{m}_{i,n+1}-2\overline{C}^{m}_{i,n}\right| + \frac{\left|\overline{C}'^{m+1}_{i,n}-\overline{C}''^{m+1}_{i,n}\right|}{H_n/W+H_n^P/W} \tag{24}$$

for Case b can be obtained.

It can be seen from both of the above two cases that, if H_n or H_n^P is large relative to W, $\left|\overline{C}_{i,n}^{m+1}-\overline{C}_{i,n}^{m}\right|$ is small, which means that one pulse can only cause little change

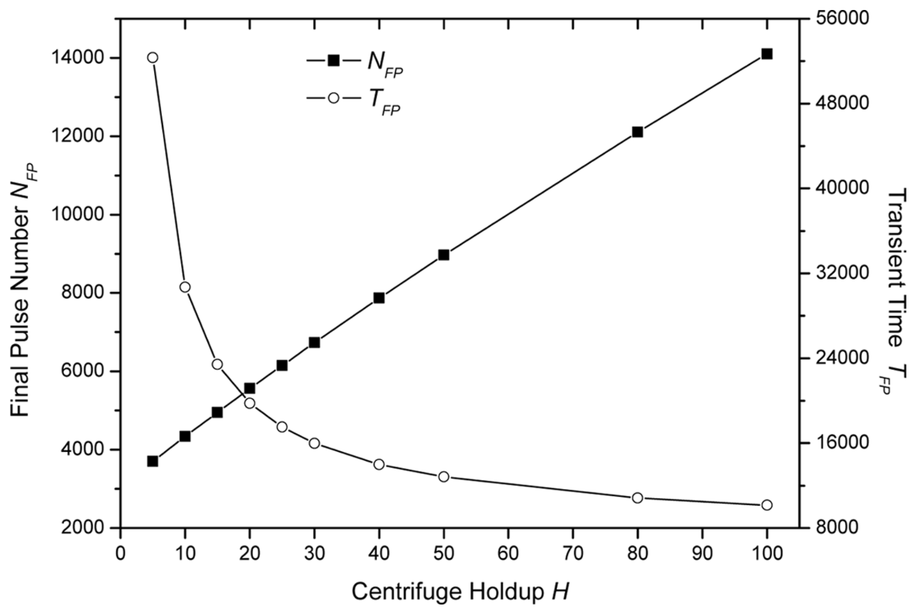

FIG. 11. The final pulse number and transient time for different centrifuge holdups ($N=11$, $N_F=5$, $H^P=20$, $F=1$, $P=0.2$).

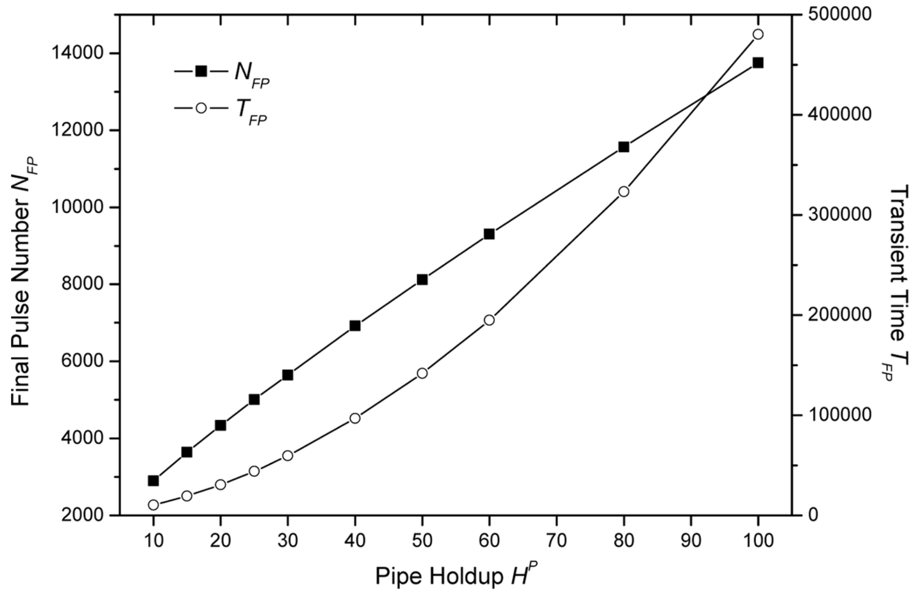

FIG. 12. The final pulse number and transient time for different pipe holdups ($N=11$, $N_F=5$, $H=10$, $F=1$, $P=0.2$).

in concentrations. Then, the concentration needs more pulses to reach the steady state. Besides, the waste here is proportional to the feed. So in the cases when the centrifuge holdup or the pipe holdup is large relative to the feed, the final pulse number is large.

Figures 13 and 14 are the integrative results of Fig. 10 to Fig. 12. In Fig. 13, the three curves almost lap over each other and the final pulse number increases almost linearly with the ratio of the total holdup in centrifuges and pipes of each stage to the feed, i.e., $(H+H^P)/F$. The three curves in Fig. 14 indicate that the transient time increases approximately linearly with the ratio $(H^P)^2/(FH)$.

One sees that the ratios $(H+H^P)/F$ and $(H^P)^2/(FH)$ play decisive roles in the transient process of reaching steady state for pulse cascades. If wanting fewer pulses and shorter time to reach steady state, these two ratios

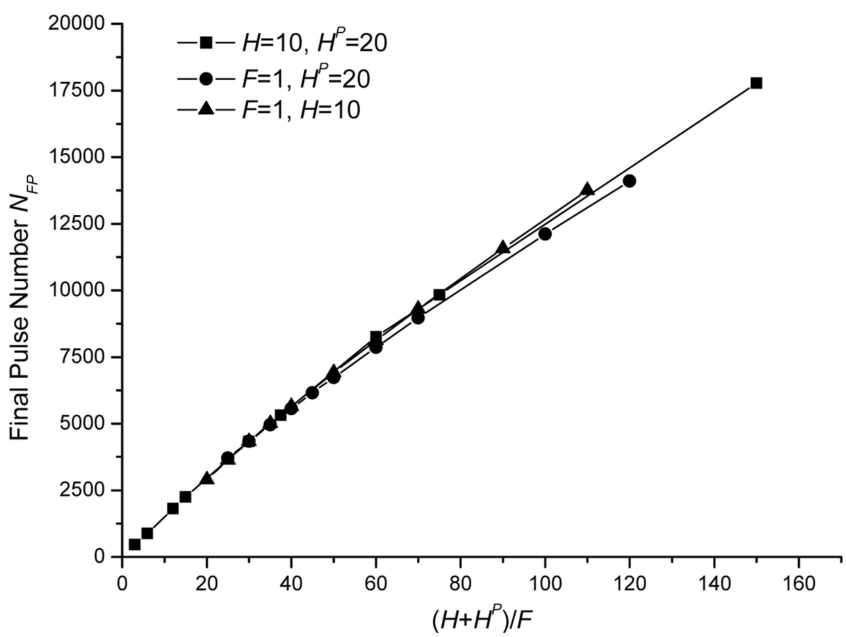

FIG. 13. The final pulse number vs. the ratio of the holdups to the feed.

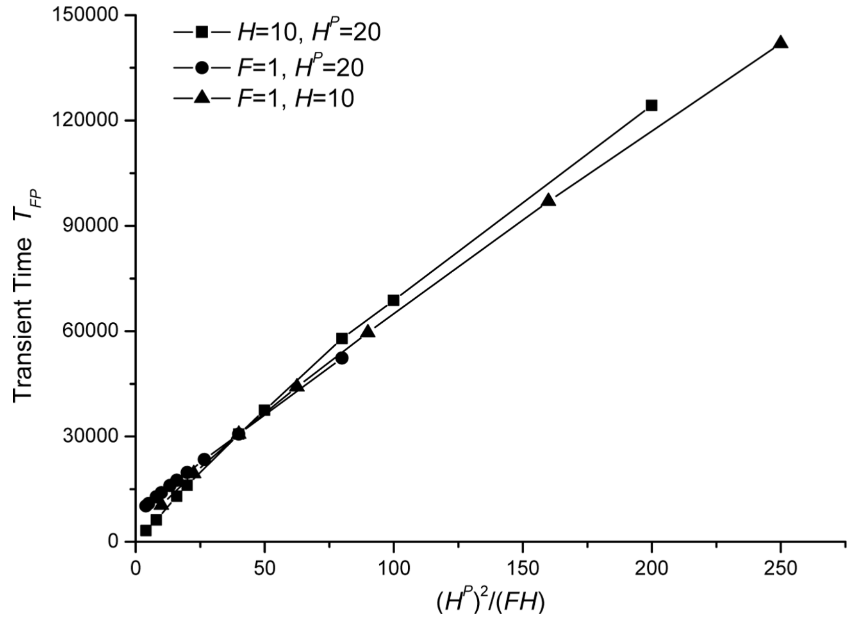

FIG. 14. The transient time vs. the ratio of $(H^P)^2/(FH)$.

should both be small, which leads to careful choosing of the magnitudes of the feed, the centrifuge holdup and the pipe holdup.

CONCLUSIONS

Pulse cascades work in a pulsant manner like a sequence of pulses. One pulse consists of two operation states—the closed state and the open state. There are no steady and continuous flows because of the alternating operation between the two states. A pulse cascade whose pipe holdups of all stages are constant is referred to as a square pulse cascade. Transient processes always exist in pulse cascades. The calculations for square pulse cascades demonstrate that the method used here is applicable in analyzing the transient processes, and lead to the following conclusions:

For square pulse cascades without feed and withdrawals, the main factors influencing the transient process are the cascade length and the ratio of the centrifuge holdup to the pipe holdup H/H^P. The longer the cascade is, the larger the final pulse number is and the longer the transient time is. The larger the ratio H/H^P is, the larger the final pulse number is, but the shorter the transient time is.

For square pulse cascades with feed and withdrawals, the factors influencing the transient process include the feed, the centrifuge holdup, and the pipe holdup in addition to the cascade length. For cascades with prescribed lengths, the final pulse number increases almost linearly with the ratio $(H+H^P)/F$ of the total holdup in the centrifuges and pipes of every stage to the feed, and the transient time increases approximately linearly with the ratio $(H^P)^2/(FH)$.

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